

# Dynamic Principal Component CAW Models for High-Dimensional Realized Covariance Matrices

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# Outline

Introduction

The DPC-CAW Model

Empirical Application

Conclusion

## Realized Covariance Models

The increasing availability of high-frequency data has led to a growing attention of the financial econometrics literature to the modeling of time-series of realized covariance matrices.

## Realized Covariance Models

The increasing availability of high-frequency data has led to a growing attention of the financial econometrics literature to the modeling of time-series of realized covariance matrices.

- **Gourieroux, Jasiak and Sufana (JoE, 2009) - Wishart Autoregressive Process.**
- **Jin and Maheu (JoFE, 2012) - Wishart-RCOV-A(K) model.**
- **Jin and Maheu (JoE, 2016) - Bayesian semiparametric modeling of RCs**
- **Noureldin, Shephard and Sheppard (JAE, 2012) - HEAVY Models**
- **Golosnoy, Gribisch and Liesenfeld (JoE, 2012) - Conditional Autoregressive Wishart (CAW) Models.**
- **Bauwens, Storti and Violante (2012, 2014, 2016) - Realized DCC (ReDCC) Models**

## Issues of existing RC Models

- Applications in high-dimensional settings are complicated if not impossible (*curse of dimensionality*)
- Empirical applications therefore do not exceed the 10-dimensional case. (except for the ReDCC specifications)
- Realistic portfolios typically consist of hundreds of assets which makes high-dimensional covariance matrix forecasting an important field of research.

## Our Contribution

- We propose the **Dynamic Principal Component (DPC) CAW model** for time-series of high-dimensional realized covariance measures, based on the DPC-GARCH model of Aielli and Caporin (2015)
- Bias and possible inconsistency of a multistep estimation procedure (similar to multistep DCC-GARCH estimation) assessed in an extensive simulation experiment.
- Forecasting ability assessed via the Model Confidence Set (MCS) of Hansen et al. (2011).
- In-sample fit for various model-order settings examined via the BIC information criterion.

## Realized Covariances Measures

Realized covariance matrix:

$$RC_t = \sum_{j=1}^m r_{j,t} r'_{j,t}, \quad (1)$$

where  $r_{j,t} = y((t-1) + j/m) - y((t-1) + (j-1)/m)$  is the  $j$ 'th intraday return vector on day  $t$ .

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- $RC_t$  is a well-known consistent nonparametric ex-post estimate of the 'true' integrated covariance matrix.
- If the data contains microstructure noise, jumps or non-synchronous trading, several alternatives, such as the multivariate realized kernel of Barndorff-Nielsen et al. (2011) can be employed.

## DPC-CAW Specification Overview

CAW distributional assumption:

$$R_t | \mathcal{F}_{t-1} \sim \mathcal{W}_n(\nu, S_t/\nu), \quad \Rightarrow E[R_t | \mathcal{F}_{t-1}] = S_t \quad (2)$$

DPC scale matrix spectral decomposition:

$$S_t = L_t D_t L_t' \quad (3)$$

Eigenvector process:

$$Q_t = (1 - a - b) L D L' + a R_{t-1} + b Q_{t-1}, \quad (4)$$

$$Q_t = L_t G_t L_t' \quad (5)$$

Eigenvalue process:

$$d_{i,t} = (1 - \alpha_i - \beta_i) d_i + \alpha_i (e_i' L_t' R_t L_t e_i) + \beta_i d_{i,t-1} \quad (6)$$

# The Conditional Autoregressive Wishart (CAW) Framework

$$R_t | \mathcal{F}_{t-1} \sim \mathcal{W}_n(\nu, S_t/\nu)$$

- The realized covariance measure  $R_t$  is assumed to follow a central Wishart distribution with  $\nu > n$  degrees of freedom, and symmetric, positive definite  $n \times n$  scale matrix  $S_t/\nu$ .
- $S_t$  can follow any kind of dynamic autoregressive specification for symmetric, p.d. matrices.

## Scale Matrix $S_t$

$S_t$  is defined by an eigenvector ( $L_t$ ) and an eigenvalue ( $D_t$ ) process.

$$S_t = L_t D_t L_t'$$

- $D_t = \text{diag}(d_{1,t}, d_{2,t}, \dots, d_{n,t})$  are the eigenvalues; the columns of  $L_t$  are the corresponding orthonormal eigenvectors.
- Dynamic extension of the prominent orthogonal GARCH (OGARCH) model of Alexander (2001).

## Eigenvector Process

The auxiliary process is defined as a scalar BEKK recursion for realized covariance measures.

$$\begin{aligned}Q_t &= (1 - a - b) S + a R_{t-1} + b Q_{t-1}, \\Q_t &= L_t G_t L_t' .\end{aligned}$$

- The scalars  $a$  and  $b$  and the intercept matrix  $S$  are parameters to be estimated.

### Assumption 1

*The eigenvalues in a spectral decomposition are arranged in strictly decreasing order.*

### Assumption 2

$0 \leq a, 0 \leq b, a + b < 1$ ;  $S$  and  $Q_0$  are positive definite.

## Eigenvalue Process

The eigenvalues  $d_{i,t}$  are assumed to follow  $n$  independent GARCH-type recursions:

$$d_{i,t} = (1 - \alpha_i - \beta_i)d_i + \alpha_i g_{i,t-1} + \beta_i d_{i,t-1}, \quad i = 1 \dots n$$

where  $g_{i,t} = e_i' L_t' R_t L_t e_i$  and  $e_i$  is a vector of zeros with a one on the  $i$ 'th position.

- $d_i$  are the eigenvalues of the intercept matrix  $S = LDL'$  of the eigenvector process.
- $g_{i,t}$  is the  $i$ 'th diagonal element of the random matrix  $L_t' R_t L_t$ .

$$g_{i,t} | \mathcal{F}_{t-1} \sim \text{Gamma}(\nu/2, 2d_{i,t}/\nu). \quad (7)$$

### Assumption 3

$$0 \leq \alpha_i, 0 \leq \beta_i, \alpha_i + \beta_i < 1, 0 < d_{0,i} \quad \forall i.$$

## Properties

$$\begin{aligned}
 E[g_{i,t}|\mathcal{F}_{t-1}] &= E[e_i' L_t' R_t L_t e_i | \mathcal{F}_{t-1}] = e_i' L_t' S_t L_t e_i \\
 &= e_i' L_t' L_t D_t L_t' L_t e_i = e_i' D_t e_i \\
 &= d_{i,t}.
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 E[d_{i,t}] &= (1 - \alpha_i - \beta_i) d_i + \alpha_i E[g_{i,t-1}] + \beta_i E[d_{i,t-1}] \\
 &= (1 - \alpha_i - \beta_i) d_i + \alpha_i E[E[g_{i,t} | \mathcal{F}_{t-1}]] + \beta_i E[d_{i,t-1}] \\
 &= (1 - \alpha_i - \beta_i) d_i + \alpha_i E[d_{i,t-1}] + \beta_i E[d_{i,t-1}] \\
 \Rightarrow E[d_{i,t}] &= d_i.
 \end{aligned} \tag{9}$$

Since the  $d_i$  are arranged in decreasing order (Assumption 1):

$$\Rightarrow E[d_{1,t}] > E[d_{2,t}] > \dots > E[d_{n,t}]. \tag{10}$$

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## Quasi-Likelihood Function

The quasi-likelihood (QL) function obtains as:

$$\mathcal{L}^*(\psi) = \sum_{t=1}^T -\frac{1}{2} [\ln |S_t(\psi)| + \text{tr}(S_t(\psi)^{-1}R_t)], \quad (11)$$

where  $\psi = (\text{vech}(S)', a, b, \{\alpha_i, \beta_i\}_{i=1}^n)'$  summarizes the parameters for the  $Q_t$  and  $d_{i,t}$  recursions.

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- The score vector is a martingale difference sequence, which implies the quasi-likelihood interpretation.
- $\mathcal{L}^*(\psi)$  is also the QL function of MGARCH models.

## Three-Step Estimation - The DPC estimator

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2. Conditional on step 1. estimate  $(a, b)'$  by fitting a scalar CAW model to  $\{R_t\}$ , essentially assuming  $R_t | \mathcal{F}_{t-1} \sim \mathcal{W}_n(\nu, Q_t/\nu)$ , with  $S \stackrel{!}{=} \hat{S}$ . Recover  $\hat{Q}_t$  to calculate  $\hat{g}_{i,t}$  for  $i = 1, \dots, n$ ;

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3. Conditional on 1. and 2. estimate  $\{\alpha_i, \beta_i\}_{i=1}^n$  via univariate QML based on Eqs. (6) and (7) separately  $\forall i$ .

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The simulation experiment shows:

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- Low to moderate biases in low ARCH / low persistence environments.
- In high ARCH / high persistence environments distortions occur which mainly affect the eigenvalues  $d_i$  for  $i > 3$ . These eigenvalues are rather low in absolute value.
- Hence biases are not expected to significantly affect the forecasting performance of the DPC-CAW model.

## Empirical Application - Data

- 100 stocks selected by liquidity from the S&P 500
- Realized covariance measures computed from one-minute intraday asset returns by the microstructure-noise and jump robust multivariate realized kernel method of Barndorff-Nielsen et al. (2011)
- The (co)variance processes are highly persistent, skewed to the right, leptokurtic and tend to move parallel to each other.
- January 1, 2002 to December 31, 2014, covering 3271 trading days

## Estimation Results - BIC

		Order of eigenvector process							
		(1,0)	(1,1)	(2,1)	(1,2)	(2,2)	(3,2)	(2,3)	(3,3)
Order of eigenvalue process	(1,0)	-2.5515	-2.7671	-2.0818	-2.4645	-2.0505	-2.0298	-2.0460	-2.0998
	(1,1)	-2.6289	-2.8342	-2.1779	-2.5363	-2.1531	-2.1402	-2.1502	-2.1892
	(2,1)	-2.6287	-2.8340	-2.1778	-2.5361	-2.1531	-2.1404	-2.1503	-2.1890
	(1,2)	-2.6304	-2.8361	-2.1793	-2.5377	-2.1546	-2.1418	-2.1517	-2.1905
	(2,2)	-2.6303	-2.8360	-2.1793	-2.5376	-2.1546	-2.1420	-2.1518	-2.1904
	(3,2)	-2.6301	-2.8358	-2.1791	-2.5374	-2.1544	-2.1418	-2.1516	-2.1902
	(2,3)	-2.6311	-2.8371	-2.1801	-2.5384	-2.1553	-2.1428	-2.1525	-2.1911
	(3,3)	-2.6309	-2.8370	-2.1799	-2.5382	-2.1552	-2.1426	-2.1524	-2.1909
	(4,3)	-2.6308	-2.8368	-2.1798	-2.5380	-2.1550	-2.1425	-2.1522	-2.1908
	(3,4)	-2.6314	-2.8374	-2.1801	-2.5384	-2.1554	-2.1428	-2.1526	-2.1911
	(4,4)	-2.6313	-2.8372	-2.1800	-2.5383	-2.1553	-2.1427	-2.1524	-2.1910
	(5,4)	-2.6312	-2.8371	-2.1798	-2.5381	-2.1551	-2.1426	-2.1523	-2.1908
	(4,5)	-2.6313	-2.8373	-2.1800	-2.5382	-2.1553	-2.1428	-2.1525	-2.1910
	(5,5)	-2.6312	-2.8372	-2.1799	-2.5381	-2.1552	-2.1426	-2.1524	-2.1908
	HAR	-2.6280	-2.8341	-2.1773	-2.5357	-2.1525	-2.1397	-2.1497	-2.1883

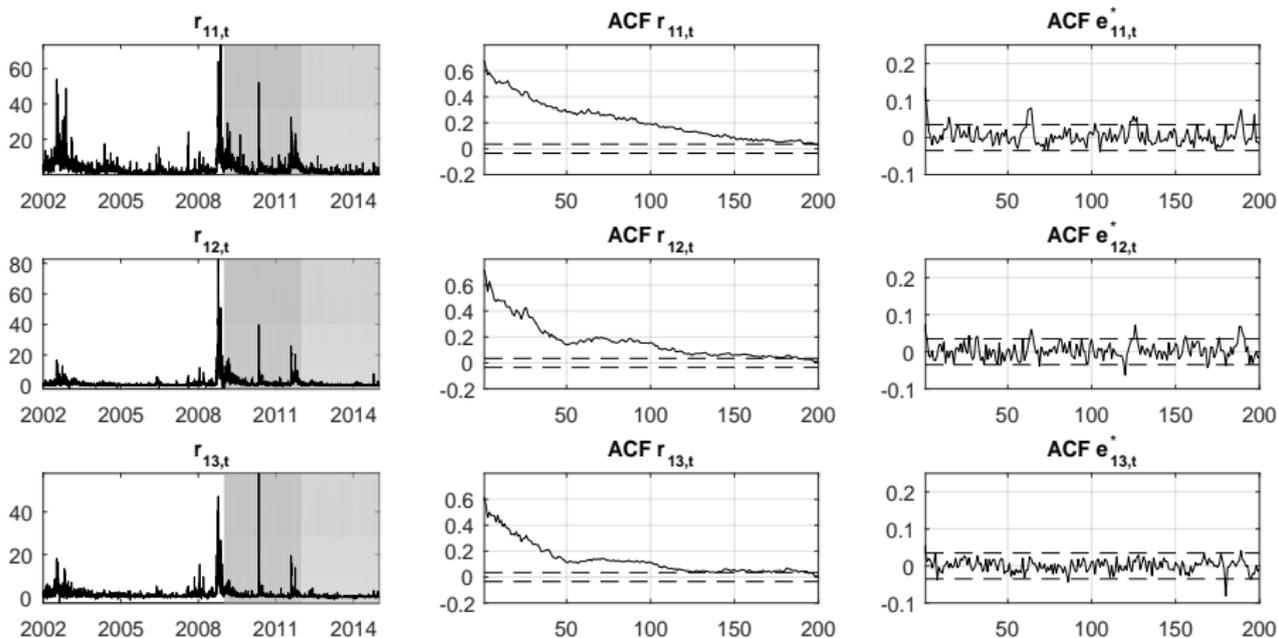
**Table:** BIC information criteria for various lag-order constellations. BIC values:  $\times 10e7$ . Models are estimated using the 3-step estimation approach. The BIC is evaluated at the full (one-step) likelihood.

## Estimation Results - DPC-CAW(1,1)-(3,4)

Eigenvalue Process								
	$\alpha_{i,1}$	$\alpha_{i,2}$	$\alpha_{i,3}$	$\beta_{i,1}$	$\beta_{i,2}$	$\beta_{i,3}$	$\beta_{i,4}$	$\sum_{\ell=1}^p \alpha_{i,\ell} + \sum_{\ell=1}^q \beta_{i,\ell}$
Median	0.311	0.074	0.000	0.132	0.068	0.135	0.135	0.978
Min.	0.025	0.000	0.000	0.000	0.000	0.000	0.000	0.947
Max.	0.492	0.180	0.116	0.519	0.348	0.373	0.378	0.987
Eigenvector Process								
	$a$	$b$	$a + b$					
	0.035	0.962	0.997					

**Table:** Summary of parameter estimates obtained by the DPC estimator for the 100-dimensional data-set described in Section ?? and the BIC selected model order (3,4)-(1,1).

# Estimation Diagnostics - DPC-CAW(1,1)-(3,4)



**Figure:** Realized (co)variance plots and sample autocorrelation functions (ACFs).  
Left panel: Sample of realized variances and covariances. Middle panel: Sample ACFs of realized (co)variances with 95% confidence bounds. Right panel: Sample ACFs and 95% confidence bounds of standardized Pearson residuals obtained from the BIC selected DPC-CAW(3,4)-(1,1)

## Forecasting Evaluation

Loss Functions:

$$L(\hat{X}, X) = \text{vech}(\hat{X} - X)' \text{vech}(\hat{X} - X)$$

(i) MSE of predicted covariance matrix:  $L(\hat{R}_t, R_t)$ ;

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- (iii) MSE of predicted correlation matrix:  $L(\hat{\rho}_t, \rho_t)$ ;

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- (iv) Variance of predicted global minimum variance portfolio (GMVP):  $V_{GMPV,t}$ ;

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- (iv) Variance of predicted global minimum variance portfolio (GMVP):  $V_{GMPV,t}$ ;
- (v) QLIKE:  $QLIKE_t = \ln |\hat{R}_t| + \text{vec}(\hat{R}_t^{-1} R_t)' \iota$ .

## Forecasting Evaluation

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- (iii) MSE of predicted correlation matrix:  $L(\hat{\rho}_t, \rho_t)$ ;
- (iv) Variance of predicted global minimum variance portfolio (GMVP):  $V_{GMPV,t}$ ;
- (v) QLIKE:  $QLIKE_t = \ln |\hat{R}_t| + \text{vec}(\hat{R}_t^{-1} R_t)' \iota$ .

The **Model Confidence Set** (MCS) by Hansen et al. (2011) contains the best model at confidence level  $1 - \alpha$ . Below we choose  $\alpha = 0.1$ .

## Competing Forecasting Models - Scalar Re-DCC

$$R_t | \mathcal{F}_{t-1} \sim \mathcal{W}_n(\nu, S_t/\nu), \quad (12)$$

$$S_t = V_t \rho_t V_t, \quad V_t = \text{diag}(\sqrt{s_{11,t}}, \sqrt{s_{22,t}}, \dots, \sqrt{s_{nn,t}}) \quad (13)$$

where  $\rho_t$  is the correlation matrix implied by  $S_t$ .

$$s_{ii,t} = \gamma_i + \sum_{k=1}^p \alpha_{k,i} r_{ii,t-k} + \sum_{l=1}^q \beta_{k,i} s_{ii,t-l}. \quad (14)$$

$$\rho_t = (1 - a - b)\bar{\rho} + aP_{t-1} + b\rho_{t-1}, \quad (15)$$

where  $P_t$  is the realized correlation matrix

$$P_t = \{\text{diag}(R_t)\}^{-1/2} R_t \{\text{diag}(R_t)\}^{-1/2}. \quad (16)$$

Three-step estimation similar to the DPC estimator:  $\bar{\rho}$  is estimated by the sample mean of realized correlation measures (“correlation targeting”).

## Competing Forecasting Models - CCC, O-CAW, EMWA

- CCC-CAW model (Re-DCC specification where  $a = b = 0$ ).
- OGARCH-CAW (DPC-CAW specification where  $a = b = 0$ ).
- Exponentially weighted moving average (EWMA) specification

$$E[R_t | \mathcal{F}_{t-1}] = (1 - \lambda)R_{t-1} + \lambda E[R_{t-1} | \mathcal{F}_{t-2}], \quad (17)$$

with preset smoothing parameter  $\lambda = 0.94$  (see J.P. Morgan, 1996).

- DPC-CAW<sub>0f</sub> model (restricting  $\alpha_i = \alpha$  and  $\beta_i = \beta$   $\forall i = 1, \dots, n$  in the DPC-CAW eigenvalue recursions).

## Forecasting - Volatile Market

Volatile Market: 01.01.2009 – 31.12.2011						
Model	(p,q)	Cov matrix	Var	Corr	GMVP $\times 10^2$	QLIKE
DPC-CAW	(1,1)	32289	8322	195.9	37.86	150.8
	(2,2)	32036	8296	195.5	37.74	150.4
	(3,3)	<b>31943</b>	8316	195.1	37.7	149.8
DPC-CAW <sub>of</sub>	(1,1)	32596	8511	193.5	37.76	147.0
	(2,2)	32262	8466	193.2	37.67	146.7
	(3,3)	32063	8450	<b>192.9</b>	<b>37.65</b>	<b>146.2</b>
Re-DCC-CAW	(1,1)	32346	8222	229.1	39.72	201.5
	(2,2)	32196	<b>8155</b>	228.8	39.59	200.4
	(3,3)	32335	8239	228.6	39.53	199.3
O-CAW	(1,1)	38834	10116	211.6	49.99	148.3
	(2,2)	38626	10112	211.2	49.97	148
	(3,3)	38528	10140	211	49.99	147.5
CCC-CAW	(1,1)	34359	8222	273.2	41.62	225.2
	(2,2)	34223	<b>8155</b>	273.2	41.54	224.7
	(3,3)	34371	8239	273.2	41.51	223.9
EWMA		37178	9823	204.9	38.9	162.9

**Table:** Mean daily forecasting losses. Grey shaded values indicate that the 90% model confidence set includes the respective model.

## Forecasting - Calm Market

Calm Market: 01.01.2012 – 31.12.2014						
Model	(p,q)	Cov matrix	Var	Corr	GMVP $\times 10^2$	QLIKE
DPC-CAW	(1,1)	1586	540.4	226.2	15.48	73.96
	(2,2)	1580	<b>539.7</b>	226.3	15.49	74.09
	(3,3)	<b>1578</b>	540.1	226.4	15.48	74.32
DPC-CAW <sub>of</sub>	(1,1)	1600	542.4	225.4	15.48	<b>66.76</b>
	(2,2)	1590	540.7	225.5	15.49	66.99
	(3,3)	1585	540.2	<b>225.4</b>	<b>15.48</b>	67.05
Re-DCC-CAW	(1,1)	1742	597.8	242.8	16.6	94.67
	(2,2)	1732	593.3	242.7	16.59	94.37
	(3,3)	1729	592.8	242.6	16.59	94.10
O-CAW	(1,1)	1756	609.4	243.9	20.08	110.16
	(2,2)	1750	608.2	244.2	20.08	110.43
	(3,3)	1749	607.6	244.5	20.08	110.59
CCC-CAW	(1,1)	1840	597.8	265.3	18.11	96.27
	(2,2)	1828	593.3	265.3	18.12	96.18
	(3,3)	1824	592.8	265.3	18.16	96.23
EWMA		1715	556.3	239.8	15.74	82.57

**Table:** Mean daily forecasting losses. Grey shaded values indicate that the 90% model confidence set includes the respective model.

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- The DPC-CAW model has particularly good forecasting properties and outperforms its competitors including DCC-CAW specifications for realized covariance measures.
- We provide an empirical application to realized covariance measures for 100 assets traded at the NYSE matrices.